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On the implications of the Bekenstein bound for black hole evaporation

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HIGHLIGHTS

- The Bekenstein bound may imply that classical spacetimes are emergent.
- Similarly, particles deemed to be elementary are "quasi particles".
- Black hole evaporation is then necessarily non unitary in any quantum gravity.
- Modified Page curves are produced for simple cases.

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ABSTRACT

We elaborate on the possible impact of the Bekenstein bound on the unitarity of black hole evaporation. As such maximal bound on the entropy of any system may be regarded as due to the existence of entities more elementary than the ordinary ones, and since at our energy scales such fundamental degrees of freedom must organize themselves into quantum fields acting on classical space-times, we then propose that both, quantum fields and geometries, are emergent phenomena stemming from the same underlying dynamics. We investigate the kinematical and model independent effects of this "quasi-particle picture" on black hole evaporation within a simple toy model, that we construct. We conclude that the information associated to the quantum fields in the "phase" before the formation of the black hole is, in general, only partially recovered in the "phase" after the black hole has evaporated. This information loss is shown to be due to the entanglement between fields and geometry. Such modifications of the Page curve

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should be regarded as common features of any theory of quantum gravity.

1. Introduction

In this paper we shall combine two general considerations regarding quantum theories of gravity to study their implications for black hole evaporation.

The first consideration is that, if the Bekenstein upper bound on the entropy of any physical system is correct, there probably exist more fundamental entities than the ones we deem to be elementary [1–3]. The second consideration is that, at our energy scales, these fundamental entities must organize themselves as quantum fields acting on classical spacetimes (our best understanding of the physics at our energy scale) by "making" both the quantum fields and the classical geometry.

The consequence of such assumptions is then the following: since the bound is reached only when the black hole is formed, and it is an *upper* bound, then and only then *all* the degrees of freedom of the fundamental entities have been excited. Hence one can think of the black hole state as a gas-like high-energy "phase", with few (one in the simplest case) macroscopic parameters characterizing all the microscopic states. When the energy is lowered (that is after the evaporation of the black hole) these fundamental entities have at their disposal a large amount of different, nonequivalent rearrangements. These are their physically nonequivalent low-energy "phases", that can only be described as specific quantum fields acting on specific geometries. Similar lessons can be learned from ordinary states of matter, made of ordinary particles. On the other hand there is a crucial difference with the usual phases of matter, namely the fact that we introduce here a democracy between spacetime and fields/particles which, in this view, both emerge from one underlying dynamics. We call this view the "quasiparticle" picture, as we shall explain below.

With this in mind, it is clearly virtually impossible that after the black hole evaporation we can retrieve the very same "phase" we had before the black hole was formed. Hence, the information associated to the quantum fields in the "phase" before the formation of the black hole is, in general, only partially recovered in the "phase" after the black hole has evaporated; the information loss is due to the entanglement between the fields and the geometry.

The structure of the paper is as follows. In Section 2 we explain more in detail the quasiparticle picture and its role in the emergence of both quantum fields and classical geometry. In Section 3 we briefly present the construction of the Page curve as well as analogous concepts and antagonist views. In order to demonstrate possible implications of our picture, in Section 4 we construct a toy model of black hole evaporation that exhibits loss of information, hence leads to a modification of the Page curve. Finally, in Section 5 we outline our conclusions.

2. The quasiparticle picture

As widely known, the entropy S of any physical system contained in a volume V, including the volume itself, is supposed to be bounded from above by the value of the Bekenstein–Hawking entropy associated to a black hole whose event horizon coincides with the boundary of V [4]

$$S \leq S_{\rm BH} = \frac{1}{4}\frac{\partial V}{\ell_P^2}, \tag{1}$$

where $\ell_P = \sqrt{\hbar G/c^3} \sim 1.6 \times 10^{-35}$ m. The generality of the original bounds for ordinary matter (i.e., when gravity is not included) posited by [5] is the subject of intense investigation and debates [3,6]. Nonetheless, it is widely accepted that for black holes the upper bound is saturated.

By the number of degrees of freedom N of a quantum physical system we mean *the number of bits of information necessary to describe the generic state of the system*. In other words, N is the logarithm of $\mathcal{N}$, the dimension of the Hilbert space of the quantum system. In the extreme case of a black hole $\mathcal{N} = e^{S_{\rm BH}}$. Hence, formula (1) means (i) that in nature the information contained in any volume V

cannot exceed 1 bit every 4 Planck areas of the boundary of V, and (ii) that *if and only if* a black hole is formed, the degrees of freedom of the hypothetical fundamental entities are most excited (see, e.g., [7]).

These fundamental entities cannot fully coincide with the particles customarily thought of as elementary (electrons, neutrinos, photons, etc.) for two reasons. One is that, if it were so, we would reach the bound with ordinary matter, and this does not happen. For instance, when gravity is not considered, the Bekenstein bound for ordinary matter (in its spherical declination) reads [5,3]:

$$S_{\text{matter}} < 2\pi \frac{RE}{\hbar c}, \tag{2}$$

where E is the total energy/mass of the system, and R is the radius of he smallest sphere encompassing the system. The second reason why the fundamental entities could be still unknown particles, is that gravity must be included in the counting of the fundamental degrees of freedom because the saturation of such bound only happens when gravity becomes as important as (or even more important then) the other interactions, to the extent of reaching the extreme case of the black hole. For instance, for the Schwarzschild black hole $E = Rc^4/(2G)$, hence the bound (2) becomes

$$S \leq 2\pi \frac{RE}{\hbar c} = \pi R^2 \frac{c^3}{\hbar G} = \frac{1}{4} \frac{\partial V}{\ell_P^2}, \tag{3}$$

where $\partial V = 4\pi R^2$ is the area of the event horizon, hence we get back the expression (1).

Although we do not know the dynamics generating the fundamental degrees of freedom, such dynamics needs be such that the emergent behavior at typical wavelengths much bigger than the Planck length ℓ_P is that of continuum quantum fields acting on a continuum classical spacetime. That is, at low resolution we have quantum field theory (QFT) in curved spacetime, hence both (classical) geometry and (quantum) fields are emergent entities.

On the one hand, the idea that gravity is an emergent phenomenon arising from more fundamental degrees of freedom is not new and goes back to Sakharov [8,9]. Presently there exist many particular models describing how gravity could emerge. The common feature of these models is to consider some kind of underlying discrete lattice. A striking fact that crystals with defects can give rise to emergent non-Euclidean geometry has been employed in the cosmological "world crystal model" [10]. It was proposed in [11] that the classical properties of the space-time might emerge from the quantum entanglement between the actual fundamental degrees of freedom. A specific model along these lines has been proposed recently in [12]. An interesting feature of this model is the possibility to recover the ER=EPR conjecture [13,14]. In quantum gravity [15,16], fundamental degrees of freedom and their interactions are represented by a complete graph with dynamical structure. For more approaches, see, e.g., [17–23].

On the other hand, emergent, nonequivalent descriptions of the same underlying dynamics are a built-in characteristic of QFT [24], both in its relativistic regime [25] (hence deemed to be fundamental) and in its nonrelativistic regime [26] (e.g., in condensed matter). Indeed, it is well recognized by now that, in general, the quantum vacuum has a nontrivial structure [27] with nonequivalent quantum mechanical sectors or "phases". This complexity is understood in QFT as due to the infinite number of degrees of freedom and/or to the nontrivial topology of the system, such as the presence of topological defects [28].

On the mathematical level these features are the manifestation of the failure of the Stone–von Neumann theorem [29,30] that holds only for quantum mechanical systems of finite degrees of freedom and trivial topology [31]. This failure leads to the existence of different, unitarily inequivalent representations of the field algebra. That is, for a given dynamics one should expect several different Hilbert spaces, representing different "phases" of the system with distinct physical properties, and distinct excitations playing the role of the elementary excitations[1] for the given "phase" [34], but whose general character is that of the quasiparticles of condensed matter [35,36].

[1] In fact, the concepts of elementary and collective excitations are interchangeable in theories where electromagnetic duality is at play [32,33].

In condensed matter examples are many. From the Cooper pairs of type II superconductors [37,38] that are bosonic quasiparticles emerging from the basic fermionic dynamics of the electrons interacting with the lattice, to the more recently discovered quasiparticles of graphene [39] that are massless Dirac quasiparticles emerging from the dynamics of electrons propagating on carbon honeycomb lattices, and giving raise to a continuum relativistic-like (2+1)-dimensional field theory on a pseudo-Riemannian geometry.

Similarly, as well known, one finds examples also in the context of black hole physics, the one of interest here. Indeed, the vacuum of a freely falling observer in Schwarzschild's spacetime can be seen, by a static observer, as a coherent state of Cooper-like pairs, similar to that of a superconductor [40], and the Hawking radiation itself is related to the existence of distinct elementary excitations in the two frames. See the original derivation of Hawking [41,42], and also [43,44].

To show here a concrete, though general enough, realization of such nontrivial vacua in QFT, let us recall the condensate structure of the vacuum of Thermo-Field Dynamics [45], that gives a general framework that applies equally well both to the condensed matter and to the black hole systems above mentioned [45]

$$|0(\theta)\rangle = \sum_n \sqrt{w_n}|n, \tilde{n}\rangle. \tag{4}$$

Here θ is the physical order parameter (temperature, acceleration, surface gravity, magnetization, etc.) labeling the different "'phases"[26], w_n are probabilities such that $\sum_n w_n = 1$, and the states $|n, \tilde{n}\rangle$ (infinite in number) are the components of the condensate, each made of pairs of n quanta and their n thermal reservoir counterparts ($\tilde{n}$). Therefore, such vacuum is clearly an entangled state. Notice that [26]

$$\langle 0(\theta)|0(\theta')\rangle \to 0, \tag{5}$$

in the field limit, that formalizes the inequivalence we have discussed. Notice also that, if one fixes θ, there is no unitary evolution to disentangle the vacuum, as the interaction with the environment and non-unitarity are the basis for the generation and the stability of such entanglement [44].

Those degrees of freedom, though, are not the fundamental ones we are referring to in this paper, because they do not explicitly include the degrees of freedom of geometry.[2] Such extra request is suggested by the Bekenstein bound, but the general mechanism we propose is similar to the one at work already in ordinary matter.

The bound (1) does not identify the type of fundamental degrees of freedom nor their dynamics. Nonetheless, we can extract from that bound one important consequence for the process of black hole evaporation. In the standard scenario assuming unitary evolution, the information contained in the collapsing matter is scrambled inside the black hole, but is eventually fully released during evaporation. This paradigm of information conservation is manifested by the so-called "Page curve" [48] (see also [49,50]) which describes the complete information retrieval in the Hawking radiation at the final stage of the black hole evaporation. In our picture, however, the probability that after the complete evaporation the fundamental degrees of freedom reorganize just like before the collapse leading to black hole, is inversely proportional to the number of possible nonequivalent rearrangements of the fundamental degrees of freedom. Therefore, even if one demands the dynamics of the fundamental degrees of freedom to be unitary, as we shall do, one expects that the entanglement between the geometry and the quantum fields due to the reshuffling of fundamental degrees of freedom could lead to a loss of information in the Hawking radiation.

The loss of information, in the sense of evolution of a pure state into a mixed state, can have two causes. The first one is that the laws of quantum theory are indeed violated in some regimes. The second one is that only some subsystem of the universe is accessible, hence there will always be a residual entanglement of the subsystem with the inaccessible parts [51]. In our picture we do not

[2] In fact, in the case of graphene, geometries can indeed be seen as emergent [46,47]. Actually, inspired by that fact that different arrangements of the carbon atoms can give rise to the same emergent spacetime geometry, in our model we take into account the possibility that the same emergent geometry can be realized through different arrangements of the fundamental degrees of freedom. These microscopic arrangements are indistinguishable at our (low) energy level.

consider the first possibility, rather we suggest that part of the total system is always hidden: this produces entanglement between emergent fields and geometry and leads to a loss of information on the field side.

Let us conclude this section with a schematic summary of the logic behind the model we propose.

(i) We interpret the upper bound (1) as indication of the existence of finite number of fundamental degrees of freedom, fully excited (saturated bound) only for a black hole. To access these fundamental degrees of freedom, one would need resolutions of order ℓ_P (which might not be possible at all, as suggested, e.g., in [52]).

(ii) At our low-energy scale (low-resolution) we wee classical spacetimes and quantum fields, both emerging from the properties of and the interactions between fundamental degrees of freedom.

(iii) Since the bound is not reached at our energies, the particles we call elementary are in fact emergent quasiparticles.

(iv) As we shall make model-independent considerations, the nature (symmetries, type of interaction, etc.) of the fundamental degrees of freedom is not specified here. In other words, we do not construct here a specific model of quantum gravity, but try to convey general considerations. Nonetheless, we do take the view that, being discrete entities, the fundamental degrees of freedom must arrange themselves into discrete structures.

(v) There are, in general, different configurations of the fundamental degrees of freedom which give rise to the same classical geometry. These configurations yield different numbers of degrees of freedom for the fields. Thus, even if the classical geometries (low-energy limit) before the formation of the black hole and after its evaporation are the same, the emerging quantum fields will, in general, be different and live in different Hilbert spaces.

(vi) Even though, for simplicity, we assume unitary evolution on the fundamental level, the rearrangement of the fundamental degrees of freedom during the evaporation process leads to an entanglement between the emerging geometry and the emerging fields, thereby producing a loss of information on the field side.

3. Page curve

Our primary motivation is to address the black hole information paradox, i.e., the problem of the apparent loss of information during the process of a black hole evaporation. There are many proposals how to resolve this paradox. There are arguments that in the presence of gravity, and especially in the presence of a black hole, we have to expect some modifications of the quantum theory and, perhaps, deviations from unitary evolution at the fundamental level. From this perspective, there is no paradox in losing information during the formation of a singularity and the subsequent evaporation of the back hole [53,54], because the underlying theory does not require the information conservation.

On the other hand, it has been advocated by [55] and [56] that the evolution is always unitary and the information loss is prohibited. These arguments rely on the holographic principle, string theory models of black hole evaporation and the paradigm of the black hole complementarity. Another confirmation of information conservation has been provided by [57], who employed the quantum perturbations of the event horizon and the AdS/CFT correspondence in order to argue that information can, in fact, escape from the black hole.

There have been also arguments that it is impossible to reconcile the unitary evolution, the principle of equivalence and the low energy emergent quantum field theory. These arguments have been embodied in the controversial "firewall paradox" introduced in [58]. Very recently, it was proposed in [59] how to avoid the firewall paradox by appropriate identification of the antipodal points of the event horizon.

A more conservative approach to the problem has been adopted by Page and it is based on purely quantum mechanical considerations. Following [60], one considers the splitting of a Hilbert space H into a bipartite system, $\mathsf{H} = \mathsf{H}_A^m \otimes \mathsf{H}_B^n$, where superscripts m and n indicate the dimension of corresponding Hilbert space, so that $\dim \mathsf{H} = m\,n$. Next, one chooses an arbitrary fixed state $|\psi_0\rangle \in \mathsf{H}$ and a random unitary matrix U; then $U|\psi_0\rangle$ is a random state in H. To such state we associate the

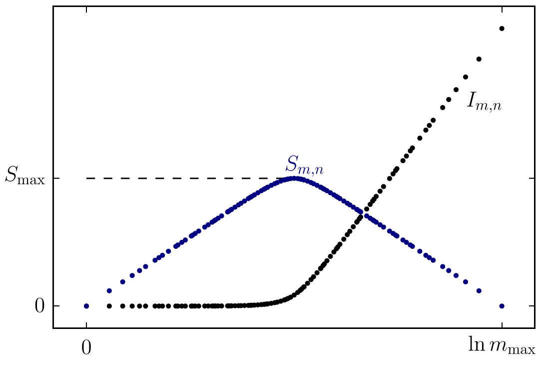

Fig. 1. Field entanglement entropy in the unitary picture (Page curve). The unitary evolution of the entanglement entropy between the field in the black hole and the field radiated out of it via the Hawking phenomenon, along with the information contained in the Hawking radiation. On the horizontal axis is the dimension of the Hilbert space of the emitted radiation. The initial and final points of this curve are the ones we are able to address in our model. (Picture adapted from [48].).

density matrix $\rho_A(U)$, by tracing out the subsystem B, and the corresponding entanglement entropy $S_{m,n}(U)$. Averaging through U we get the average entanglement entropy of the subsystem A,

$$S_{m,n} = \left\langle S_{m,n}(U) \right\rangle \,, \tag{6}$$

and the average information contained in A,

$$I_{m,n} = \ln m - S_{m,n}. \tag{7}$$

For mathematical details of this construction see the original paper [60], and also [49]. Page conjectured – and it was later proved in [61] –that the average information is

$$I_{m,n} = \ln m + \frac{m-1}{2n} - \sum_{k=n+1}^{mn} \frac{1}{k} \,, \tag{8}$$

for $m < n$.

These results are applied to the black hole evaporation problem in [48]. It is assumed that the evolution of the collapsing matter to produce a black hole and the subsequent evaporation of that black hole is a unitary process, and hence there exists a S-matrix relating the initial collapsing matter to the final state when black hole is fully evaporated and only the Hawking radiation remains. The Hilbert space of the Hawking radiation is factorized into a product as before, where the subsystem A now corresponds to the states under the horizon and the subsystem B corresponds to photons already emitted from the black hole.

When the black hole is formed, there is no Hawking radiation outside and, hence, $n = 1$ and $m = \dim \mathsf{H}$. Thus, by assumption, the entanglement entropy is trivially zero. As the black hole evaporates, dimension n increases and m decreases, while $m\,n$ is kept constant. Since the emitted photons are entangled with the particles under the horizon, entanglement entropy increases. At some stage of the evaporation (approximately half time of evaporation process) the information stored below the horizon starts to leak from the black hole, decreasing the entanglement entropy. Finally, when the black hole fully evaporates, $m = 1$ and $n = \dim \mathsf{H}$ and the entanglement entropy returns to zero. The process is shown in Fig. 1.

A natural generalization of the Page analysis is to consider tripartite system instead of a bipartite one. In [62] the authors investigate the possibility that the particles emitted by a black hole are transformed either into the Hawking radiation or into another form of matter, which can be, e.g., a remnant. In this case, even when the black hole is fully evaporated, there can still exist entanglement between these two forms of matter. Hence, the Hawking radiation does not contain the full information and it is not in a pure state (at least on average).

In the aforementioned works, no analysis of the interaction between the matter fields and the space-time geometry has been given, nor even addressed. However, in order to consider the full black hole evaporation process, one certainly cannot treat matter as the test field on a given, say Schwarzschild, background. Even assuming unitary evolution, it is the system gravity+matter which evolves unitarily, not just the matter side. Therefore, we propose the possibility that at the end of the evaporation the Hawking radiation is not in a pure state because of its entanglement with the geometry itself and it is a purpose of this paper to clarify this statement.

Thus, in a sense, we proceed analogously to Page, and it is important to stress the points where we differ. Similarly to Page, we consider what we call *fundamental* Hilbert space H, and we do not specify any particular microscopic dynamics, as we merely provide a kinematical framework which allows us to estimate the entanglement entropy. On the other hand, since we interpret both gravity and fields as emergent phenomena, we do not split H into a direct product of the two spaces (for the black hole and for the radiation), because we claim that, on the fundamental level, there is no distinction between the field and the geometry at all. Instead, we introduce *emergent* Hilbert spaces representing the states of the geometry and of the fields, and *mappings* which extract the geometrical and the matter content from the states of H. As a consequence, the loss of information, in our picture, does not require the presence of a third, unknown kind of matter like in [62], because it is due to the entanglement of the matter field with the geometry (nonetheless, our description does allow for an arbitrary number of different fields, hence naturally includes the possibility for an unknown kind of matter).

4. Model of black hole evaporation

Our goal in this section is to construct a simple kinematical model which mimics the evaporation of the black hole within the "quasi-particle" picture above presented. We consider the following idealized scenario:

1. Initially, there is a quantum field (in an almost flat space) which collapses and eventually forms a black hole of mass M_0.
2. The black hole starts to evaporate in a discrete way; for simplicity we assume that each emitted quantum of the field has the same energy ε, so that $M_0 = N_G\,\varepsilon$ for some integer N_G.
3. At the end of the evaporation, the space becomes almost flat again and the field is in excited state with N_G quanta.

We assume:

1. There exists a *fundamental Hilbert space* H. That is the Hilbert space of the fundamental degrees of freedom of the total system, i.e., black hole, radiation and space outside the black hole. Since here we focus on a finite region accessible to a generic observer and big enough to contain the black hole at initial time and the emitted radiation at a later time, H here is finite-dimensional;
2. For a specific observer at low-energy scale, the states of H appear as classical *spatial* geometry and quantum fields propagating on it.
3. There are states in H which represent the same classical geometry but are microscopically different.
4. In general, there is exchange of the number of degrees of freedom between the fields and geometry.

In this model, we introduce a space of classical geometries representing spatial slices of space-time containing a black hole of a given mass $M^{(a)} = a\,\varepsilon$. That is, we introduce an orthonormal set of the states

$$|g^{(a)}\rangle, \qquad a = 0, 1, \ldots, N_G - 1, \tag{9}$$

where N_G is therefore the number of geometries allowed in our model. For convenience, we introduce the *Hilbert space of classical geometries* H_G as the linear span of the states (9) and define the "mass operator" M by

$$\mathsf{M}|g^{(a)}\rangle = M^{(a)}|g^{(a)}\rangle \equiv \varepsilon\, a\, |g^{(a)}\rangle. \tag{10}$$

An operator of this kind should represent the possibility of measuring geometric properties of the space, such as the three-dimensional metric, as seen by a specific observer. The assumption that the geometry of the space is a result of some coarse-graining procedure associated with a specific observer means there is some mapping $\mathsf{P}_\mathsf{G} : \mathsf{H} \mapsto \mathsf{H}_\mathsf{G}$ which assigns to a microscopic state in H corresponding classical geometry or an appropriate superposition of such geometries. This is analogous to the "emergence map" introduced in [63].

Similarly, we shall assume the existence of some mapping $\mathsf{P}_\mathsf{F} : \mathsf{H} \mapsto \mathsf{H}_\mathsf{F}$ which extracts the "field content" of a state in H. Then, H_F can be, e.g., an appropriate Hilbert (Fock) space representing the states of the fields; concrete definitions will depend on the particular theory of quantum gravity. Schematically, the states of the fundamental Hilbert space H can be interpreted as states with some classical geometry via the mapping P_G, and with some state of the quantum field via the mapping P_F:

$$\begin{tikzcd} & {|\psi\rangle \in \mathsf{H}} \arrow[dl, "\mathsf{P}_\mathsf{G}"'] \arrow[dr, "\mathsf{P}_\mathsf{F}"] & \\ {|g^{(a)}\rangle \in \mathsf{H}_\mathsf{G}} & & {|\phi\rangle \in \mathsf{H}_\mathsf{F}} \end{tikzcd}$$

After introducing these mappings, one can label the states in H by the values of the coarse-grained quantities, i.e., $|\psi\rangle = |g^{(a)}, \phi\rangle$.

For simplicity we assume that any state of H can be interpreted in such a way, although in reality this is much more complicated: classical geometries are expected to be very special superpositions of basis states with no classical analogues. Since we are not building a specific model of quantum gravity, we ignore this complication. On the other hand, one can argue that among the states corresponding to definite classical geometries one can choose a subset of (sufficiently distinct) states which are approximately orthogonal and consider only a subspace of H generated by this (approximately) orthonormal set.

In Page's picture described in Section 3 he considers splitting of the Hilbert space representing the states of the field into "inside" and "outside" part with respect to the horizon of the black hole. In our model we wish to implement the idea that the geometry and its fundamental degrees of freedom must be brought into the picture, so that one should split the fundamental space H into a direct product of "geometrical" and "field" part. However, for our argument it is essential to entertain the possibility that the distribution of the microscopic degrees of freedom between the geometry and the fields is not fixed and can change during the evolution of the system.

The following simple model will serve just as a useful visualization and to provide a terminology convenient for the subsequent construction. However, the construction itself does not rely on such visualization. Let there be a certain number N of fundamental degrees of freedom in the sense explained in the Introduction. The states of each fundamental degree of freedom form a d-dimensional Hilbert space, so that the Hilbert space of all fundamental degrees of freedom has dimension d^N. Now, the states of the fundamental degrees of freedom give rise to the notions of spatial geometry (distance, topology, dimension) and of quantum fields. We think of the set of all fundamental degrees of freedom as distributed among the vertices of a graph and their links. More specifically, suppose that the fact that there is some geometrical relation (e.g., the distance) between two vertices can be represented as a link between corresponding vertices of the graph and a quantitative measure of such relation is represented by a weight (or a set of weights) of the link. Thus, one could interpret the geometry as encoded in the states of all links in the graph. However, in order to keep the total degrees of freedom constant, the vertices have to "offer" some of their degrees of freedom to form the Hilbert space H_G corresponding to the states of the links which are interpreted as the state of the geometry. Then, the remaining fundamental degrees of freedom can be represented as excitation states of the vertices of the graph and they form a Hilbert space H_F whose elements are interpreted as the states of the emergent field. So, the state of the entire graph is an element of the Hilbert space $\mathsf{H}_\mathsf{G} \otimes \mathsf{H}_\mathsf{F}$ of dimension d^N. The point is that it is the *topology* of the graph (by which we mean simply a specific distribution of the links, ignoring their weights and the states of the vertices) which dictates how the available fundamental degrees of freedom are distributed between the fields and the geometry. During a standard, "nonviolent" evolution, we might expect that the topology of the lattice does not change, but as the black hole and singularity form, significant changes of the topology happen, implying both topological and causal changes in the emergent spatial geometry and possible deviations from standard QFT on curved space-time in the following sense: the change of the topology of the lattice means reshuffling of the fundamental degrees of freedom between the geometry and the fields, so that the structure of the new graph is $\mathsf{H}'_{\mathsf{G}} \otimes \mathsf{H}'_{\mathsf{F}}$; the fields now live in a Hilbert space H'_{F} of different dimension than H_{F}. In this case we have to expect the deviations from the unitary evolution on the emergent field side, although the underlying evolution of microscopic degrees of freedom is purely unitary.

We do not stick to this oversimplified picture in which the weights of the links are related directly to the metric and the states of the vertices are related directly to the states of the fields. We shall, however, stick to the idea that there are several ways how the fundamental degrees of freedom are reshuffled between the fields and the geometry and, in addition, there might exist different microscopic configurations which, on the emergent level, give rise to the same coarse-grained geometry. On the emergent level it is impossible to distinguish between two such microscopic configurations but, microscopically, the two configurations differ by the number of degrees of freedom available for the fields. That is, the fields in the two cases are elements of different Hilbert spaces and, hence, the resulting field cannot be in a pure state.

4.1. Toy model

Hence, starting from the fundamental Hilbert space H, we assume it can be split into a direct sum of the subspaces $T_{(i)}$,

$$\mathsf{H} = \bigoplus_{i=1}^{N_T} T_{(i)}, \qquad \dim \mathsf{H} = N_T\, N, \tag{11}$$

where each $T_{(i)}$ has a fixed dimension N and consists of states with some specific distribution of the degrees of freedom between the geometry and the fields; in the language of the simplistic "graph model", $T_{(i)}$ is a set of states for one specific choice of the topology of the graph and hence we shall refer to $T_{(i)}$ as the set of the states with specific topology; N_T is then the number of different topologies. By assumption, each $T_{(i)}$ has a structure

$$T_{(i)} = \mathsf{H}_{\mathsf{G}}^{p_i} \otimes \mathsf{H}_{\mathsf{F}}^{q_i}, \qquad p_i\, q_i = N, \tag{12}$$

where $\mathsf{H}_{\mathsf{G}}^{p}$ ($\mathsf{H}_{\mathsf{F}}^{q}$) is a Hilbert space of dimension p (q) representing possible microscopic states of the geometry (fields).

A general state $|\psi\rangle \in \mathsf{H}$ admits the expansion adapted to the splitting of H which is in the form

$$|\psi\rangle = \bigoplus_{i=1}^{N_T} \sum_{I=1}^{p_i} \sum_{n=0}^{q_i-1} c_{In}^{(i)} |I_i\rangle \otimes |n_i\rangle, \tag{13}$$

where vectors $|I_i\rangle$ and $|n_i\rangle$ form a basis of spaces $\mathsf{H}_{\mathsf{G}}^{p_i}$ and $\mathsf{H}_{\mathsf{F}}^{q_i}$, respectively.

Let us denote by $\mathsf{P}_{(i)} : \mathsf{H} \mapsto T_{(i)}$ a projector onto the subspace $T_{(i)}$. Then, the squared norm of the state $\mathsf{P}_{(i)}|\psi\rangle$ is the probability $p_{(i)}$ of finding the system in the state with the topology $T_{(i)}$,

$$p_{(i)} = \|\mathsf{P}_{(i)}|\psi\rangle\|^2. \tag{14}$$

In general, state in $T_{(i)}$ is a state with the entanglement between the geometry and the field in the sense that its decomposition reads

$$\mathsf{P}_{(i)}|\psi\rangle = \sum_{I,n} c_{In}^{(i)} |I_i\rangle \otimes |n_i\rangle. \tag{15}$$

Associated density matrix representing the state of the field is

$$\rho_{(i)} = \mathrm{Tr}_{\mathsf{H}_{\mathsf{G}}^{p_i}} |\psi\rangle_i \langle\psi|_i, \tag{16}$$

where we first define the normalized state

$$|\psi\rangle_i = p_{(i)}^{-1/2}\mathsf{P}_{(i)}|\psi\rangle \tag{17}$$

and then trace over the degrees of freedom of the gravitational field. Corresponding entanglement entropy will be denoted by

$$S_{(i)} = -\mathrm{Tr}_{\mathsf{H}_{\mathsf{F}}^{q_i}}\,\rho_{(i)}\ln\rho_{(i)}; \tag{18}$$

$S_{(i)}$ is the entanglement entropy between the geometry and the fields for a given topology of the lattice. Since for the observer it is impossible to distinguish between different topologies of the lattice, expected value of the entanglement between the fields and the geometrical degrees of freedom will be

$$\langle S\rangle = \sum_i p_{(i)}S_{(i)}. \tag{19}$$

In our toy model we shall assume that only two topologies are possible, i.e., $N_T = 2$, and that both topologies admit the same family of classical geometries (9), i.e., we assume

$$\mathsf{P}_{\mathsf{G}}(T_{(1)}) = \mathsf{P}_{\mathsf{G}}(T_{(2)}) = \mathsf{H}_{\mathsf{G}}. \tag{20}$$

Let us fix the number of degrees of freedom for each type of the lattice to $N = 1500$ and let us set

$$\begin{aligned} T_{(1)} &= \mathsf{H}_{\mathsf{G}}^{30}\otimes\mathsf{H}_{\mathsf{F}}^{50}, \quad p_1\times q_1 = 30\times 50,\\ T_{(2)} &= \mathsf{H}_{\mathsf{G}}^{60}\otimes\mathsf{H}_{\mathsf{F}}^{25}, \quad p_2\times q_2 = 60\times 25. \end{aligned} \tag{21}$$

then we have $\dim\mathsf{H} = 3000$. Finally, assume that the maximal mass M_0 of the black hole is split into $N_G = 30$ quanta. That is, for a black hole of mass $M^{(a)} = a\,\varepsilon$ there is exactly one state in $\mathsf{H}_{\mathsf{G}}^{30}$ such that the mapping P_{G} maps it to a state $|g^{(a)}\rangle$, while in $\mathsf{H}_{\mathsf{G}}^{60}$ there are two such states. Hence, the subspaces $T_{(i)}$ are generated by the following bases:

$$\begin{aligned} T_{(1)} &= \mathrm{span}\left\{|(a)\rangle\otimes|n\rangle\right\},\\ T_{(2)} &= \mathrm{span}\left\{|(a),1\rangle\otimes|n\rangle,\,|(a),2\rangle\otimes|n\rangle\right\}, \end{aligned} \tag{22}$$

where

$$\begin{aligned} \mathsf{P}_{\mathsf{G}} &: |(a)\rangle\otimes|n\rangle\in\mathsf{H}_{\mathsf{G}}^{(30)}\otimes\mathsf{H}_{\mathsf{F}}^{(50)}\mapsto|g^{(a)}\rangle\in\mathsf{H}_{\mathsf{G}},\\ &: |(a),1\rangle\otimes|n\rangle\in\mathsf{H}_{\mathsf{G}}^{(60)}\otimes\mathsf{H}_{\mathsf{F}}^{(25)}\mapsto|g^{(a)}\rangle\in\mathsf{H}_{\mathsf{G}},\\ &: |(a),2\rangle\otimes|n\rangle\in\mathsf{H}_{\mathsf{G}}^{(60)}\otimes\mathsf{H}_{\mathsf{F}}^{(25)}\mapsto|g^{(a)}\rangle\in\mathsf{H}_{\mathsf{G}}. \end{aligned} \tag{23}$$

We shall interpret elements $|n\rangle\in\mathsf{H}_{\mathsf{F}}^{q}$ as the states of the quantum field with n particles.

A general state in H can be now written in the form

$$|\psi\rangle = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1}\alpha_{an}|(a)\rangle\otimes|n\rangle \tag{24}$$

$$+\sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1}\left(\beta_{an}|(a),1\rangle\otimes|n\rangle+\gamma_{an}|(a),2\rangle\otimes|n\rangle\right), \tag{25}$$

and the expected value of the mass of the black hole is

$$\langle M\rangle = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1}M^{(a)}|\alpha_{an}|^2 \tag{26}$$

$$+\sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1}M^{(a)}(|\beta_{an}|^2+|\gamma_{an}|^2), \tag{27}$$

and similarly for the expected number of particles $\langle n\rangle$. The probabilities for finding the lattice in the state with the topology $T_{(1)}$ and $T_{(2)}$, respectively, are

$$p_{(1)} = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} |\alpha_{an}|^2, \tag{28}$$
$$p_{(2)} = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1} \left(|\beta_{an}|^2 + |\gamma_{an}|^2\right). \tag{29}$$

In H the evolution (first formation of the black hole, then evaporation) is continuous and unitary. We start the analysis at the point when the black hole just formed and the field outside the black hole is in the ground state, i.e., in the state with zero particles. Then the evaporation starts which we mimic by prescribing the expected values of mass $\langle M\rangle$ and the expected value of number of particles $\langle n\rangle$. Since we do not know the underlying microscopic dynamics, we shall consider, similarly to Page, all states which are compatible with these expectation values.

Let us find a convenient parametrization of such states. First, since the general state (24) must be normalized,

$$\sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} |\alpha_{an}|^2 + \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1}(|\beta_{an}|^2 + |\gamma_{an}|^2) = 1 \tag{30}$$

we can set

$$p_{(1)} = \sum_{n=0}^{q_1-1} |\alpha_{an}|^2 = \cos^2\theta,$$
$$p_{(2)} = \sum_{n=0}^{q_2-1}(|\beta_{an}|^2 + |\gamma_{an}|^2) = \sin^2\theta, \tag{31}$$

where $\theta \in (0, \pi/2)$ without the loss of generality. Let us parametrize the coefficients by

$$|\alpha_{an}| = \mu_{an}\cos\theta, \tag{32}$$
$$|\beta_{an}| = \nu_{an}\sin\theta\cos\phi, \tag{33}$$
$$|\gamma_{an}| = \lambda_{an}\sin\theta\sin\phi, \tag{34}$$

where $\phi \in (0, \pi/2)$. This is analogous to introducing the Hopf coordinates on the sphere S^n. This parametrization implies

$$\sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} \mu_{an}^2 = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1} \nu_n^2 = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1} \lambda_n^2 = 1. \tag{35}$$

We also define

$$\mu_M = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} M^{(a)}\mu_{an}^2,\ \mu_N = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} n\,\mu_{an}^2 \tag{36}$$

and similarly for ν_M, ν_N and λ_M, λ_N. In this notation, the expected value of the mass of the black hole and the expected number of particles are given by

$$\langle M\rangle = \mu_M\cos^2\theta + \sin^2\theta(\nu_M\cos^2\phi + \lambda_M\sin^2\phi), \tag{37}$$
$$\langle n\rangle = \mu_N\cos^2\theta + \sin^2\theta(\nu_N\cos^2\phi + \lambda_N\sin^2\phi). \tag{38}$$

A generic state (24) now acquires the form

$$|\psi\rangle = \cos\theta \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} \mu_{an} e^{i\chi_{an}^{\alpha}} |(a)\rangle \otimes |n\rangle \tag{39}$$

$$+ \sin\theta \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1} \Big(\nu_{an} e^{i\chi_{an}^{\beta}} \cos\phi |(a), 1\rangle \otimes |n\rangle \tag{40}$$

$$+ \lambda_{an} e^{i\chi_{an}^{\gamma}} \sin\phi |(a), 2\rangle \otimes |n\rangle \Big) . \tag{41}$$

The normalized projections of $|\psi\rangle$ onto $T_{(1)}$ and $T_{(2)}$ correspond to the first and the second sum in (41), respectively, with factors $\sin\theta$ and $\cos\theta$ omitted:

$$|\psi\rangle_1 = \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_1-1} \mu_{an}\, e^{i\chi_{an}^{\alpha}} |(a)\rangle \otimes |n\rangle, \tag{42}$$

$$|\psi\rangle_2 = \cos\phi \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1} \nu_{an}\, e^{i\chi_{an}^{\beta}} |(a), 1\rangle \otimes |n\rangle$$

$$+ \sin\phi \sum_{a=0}^{N_G-1}\sum_{n=0}^{q_2-1} \lambda_{an}\, e^{i\chi_{an}^{\gamma}} |(a), 2\rangle \otimes |n\rangle. \tag{43}$$

Corresponding density matrices are

$$\rho_{(i)} = \sum_{m,n=0}^{q_i-1} c_{nm}^{(i)}\, |n\rangle\langle m|, \quad i = 1, 2, \tag{44}$$

where

$$c_{mn}^{(1)} = \sum_{a=0}^{N_G-1} \mu_{an}\, \mu_{am}\, e^{i\chi_{an}^{\alpha} - i\chi_{am}^{\alpha}}, \tag{45}$$

$$c_{mn}^{(2)} = \sum_{a=0}^{N_G-1} \left(\nu_{an}\, \nu_{am}\, e^{i\chi_{an}^{\beta} - i\chi_{am}^{\beta}} + \lambda_{an}\, \lambda_{am}\, e^{i\chi_{an}^{\gamma} - i\chi_{am}^{\gamma}} \right). \tag{46}$$

With each density matrix $\rho_{(i)}$ there is associated entanglement entropy $S_{(i)}$ given by (18), so that the average value of the entanglement entropy is (19)

$$\langle S\rangle = S_{(1)}\cos^2\theta + S_{(2)}\sin^2\theta. \tag{47}$$

For the purposes of this paper, the corresponding entanglement entropy will be calculated numerically.

4.2. *Entanglement entropy calculation*

We wish to estimate the expected entanglement entropy for a random state which has specific expected value of number of particles $\langle n\rangle$. In order to do that, we would need to solve the constraints (38) for given values $\langle M\rangle$ and $\langle n\rangle$ with respect to the parameters θ, ϕ, μ_M, μ_N, ν_M, ν_N and λ_M, λ_N. For the purposes of this paper we chose the following way of estimating the entanglement entropy.

Conditions (38) can be rewritten in the form

$$\begin{aligned} x^2 &= \frac{\mu_M}{\langle M\rangle}\cos^2\theta, & x'^2 &= \frac{\mu_N}{\langle n\rangle}\cos^2\theta, \\ y^2 &= \frac{\nu_M}{\langle M\rangle}\sin^2\theta\cos^2\phi, & y'^2 &= \frac{\nu_N}{\langle n\rangle}\sin^2\theta\cos^2\phi, \end{aligned} \tag{48}$$

$$z^2 = \frac{\lambda_M}{\langle M\rangle}\sin^2\theta\sin^2\phi,\ \ z'^2 = \frac{\lambda_N}{\langle n\rangle}\sin^2\theta\sin^2\phi,$$

where (x, y, z) and (x', y', z') are points on the unit 2-sphere. Hence, we generate two random unit 3-vectors and choose $\cos^2\theta$ and $\cos^2\phi$ randomly with the uniform distribution on the interval $(0, 1)$ and check, whether conditions

$$b\frac{\langle M\rangle\, x^2}{\cos^2\theta} \le N_G - 1, \qquad \frac{\langle n\rangle\, x'^2}{\cos^2\theta} \le q_1 - 1, \tag{49}$$

$$\frac{\langle M\rangle\, y^2}{\sin^2\theta\cos^2\phi} \le N_G - 1,\ \ \frac{\langle n\rangle\, y'^2}{\sin^2\theta\cos^2\phi} \le q_2 - 1, \tag{50}$$

$$\frac{\langle M\rangle\, z^2}{\sin^2\theta\sin^2\phi} \le N_G - 1,\ \ \frac{\langle n\rangle\, z'^2}{\sin^2\theta\sin^2\phi} \le q_2 - 1, \tag{51}$$

are satisfied, where, we recall, N_G is the number of allowed geometries and $q_1 = \dim \mathsf{H}_F^{q_1} = 30$, $q_2 = \dim \mathsf{H}_F^{q_2} = 60$ are the dimensions of Hilbert spaces for the fields corresponding to the two topologies of the lattice. If the conditions do not hold, we iterate the procedure until we find such combination of θ, ϕ, (x, y, z) and (x', y', z'). This ensures that sequences μ_{an}, ν_{an} and λ_{an} with the desired averages μ_M, μ_N, ν_M, ν_N, λ_M and λ_N exist. Then we generate such sequences. Finally, we choose the phases χ^{α}_{an}, χ^{β}_{an} and χ^{γ}_{an} randomly with uniform distribution on the interval $(0, 2\pi)$. In this way we generate a random state yielding the prescribed expectation values $\langle M\rangle$ and $\langle n\rangle$; the entropy is then calculated by means of Eqs. (18) and (46). Then we calculate the average value of entanglement entropy from 5000 runs of this procedure.

Now we consider the scenario of evaporation of the black hole. At the beginning, let the black hole have its maximal mass $M_0 = (N_G - 1)\varepsilon$, where we choose $N_G = 30$ and let there be vacuum outside the black hole. That does not necessarily mean that the Hilbert space for the field outside the black hole has dimension 1 (as it is in Page's case); indeed, in our model we have chosen the dimension to be either 50 or 25, depending on the topology of the lattice. However, since there is only one vacuum state $|0\rangle \in \mathsf{H}_F^{50}$ and only one vacuum state $|0\rangle \in \mathsf{H}_F^{25}$, in both topologies, the field is disentangled from the geometry. This can be also seen from Eq. (38) which shows that for $\langle n\rangle = 0$ we have

$$\mu_N = \nu_N = \lambda_N = 0 \tag{52}$$

which, by (36), implies

$$\mu_{an} = 0\,, \tag{53}$$

for $n > 0$, and similarly for ν_{an} and λ_{an}. Requirement $\mu_M = M_0$ then implies that the only nonzero μ_{an} is

$$\mu_{(N_G-1),0} = 1. \tag{54}$$

Then both states $|\psi\rangle_1$ and $|\psi\rangle_2$ in (43) are unentangled and we have $\langle S\rangle = 0$. Hence, our starting point coincides with the starting point of Page: expected entanglement vanishes at the beginning of the evaporation.

Now we assume that black hole starts to evaporate. We assume continuous unitary evolution of the state in H but take the "snapshots" of the system when the expected values are

$$\langle M\rangle = (N_G - 1 - k)\,\varepsilon, \quad \langle n\rangle = k, \tag{55}$$

where k acquires discrete values

$$k = 0, 1, \ldots, N_G - 1; \tag{56}$$

$k = 0$ corresponds to black hole of maximal mass $M_0 = M^{(N_G-1)} = (N_G - 1)\varepsilon$ and vacuum outside the black hole; in kth step, black hole already emitted k quanta of the field, so its mass decreased by the value $k\,\varepsilon$, while the field is in the state with k quanta outside; black hole is fully evaporated for $k = N_G - 1$ and the field is in the state with $N_G - 1$ particles.

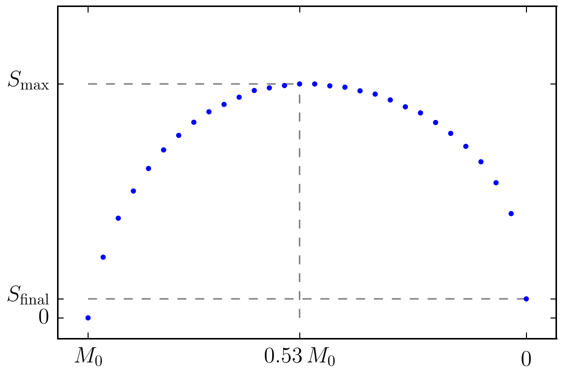

Fig. 2. Field entanglement entropy in the quasi-particle picture. Entropy of the field due to its entanglement with the geometry as a function of the decreasing mass of the evaporating black hole. The initial and final points of this curve are in exact correspondence with the initial and final points of the Page curve. Therefore, in particular, we can compare the end point here with the end point there. We see here that, at the end of the evaporation, the entanglement entropy is still finite. Even if the deviation of $\langle S\rangle$ from the pure state is small, its nonzero value signals a dramatic departure from the *information conservation* scenario. Notice also that, allowing for more microscopic realizations of the same macroscopic geometry, would in general increase the size of the final deviation.

Notice that at the end of the evaporation, the state $|\psi\rangle_1$ is disentangled again, but the state $|\psi\rangle_2$ remains entangled. Hence, the expected value of the entanglement entropy decreases but remains nonzero.

In Fig. 2 we show the entanglement entropy as the function of the discrete parameter k. Although this graph starts at the point $(M_0, 0)$ which corresponds to the same origin of the Page curve in Fig. 1, at the final stage of the evaporation the entanglement entropy does not go to zero; at this point we differ from the prediction of the Page curve. It is clear that allowing for more microscopic realizations of the same emergent geometry, i.e., more topologies, would in general increase the final deviation of $\langle S\rangle$ from the pure state value, as we now show.

The Hilbert space has the structure

$$\mathsf{H} = \bigoplus_{i=1}^{N_T} \mathsf{H}_G^{p_i} \otimes \mathsf{H}_F^{q_i}, \tag{57}$$

where N^T is the number of topologies.[3] Hilbert space $\mathsf{H}_G^{p_i}$ of dimension p_i represents the states which look like classical geometries. In the toy model, we assume that there are always N_G classical geometries available and they represent the black hole with mass $M^{(a)} = a\,\varepsilon$, where $a = 0, 1, \ldots, N_G - 1$. Each classical geometry can be realized by R_G^i microstates, so that

$$p_i = N_G\, R_G^i. \tag{58}$$

Similarly, $\mathsf{H}_F^{q_i}$ is the Hilbert space of dimension q_i representing states which appear as states of quantum field on the emergent level. Each emergent state $|n\rangle$ can be realized by R_F^i indistinguishable microstates.

In Fig. 3 we choose $N_G = 30$, i.e. the mass of the black is split into 30 quanta. We choose $N_T = 2$ (two different topologies), assume that $R_F^i = 1$ for each topology, and we plot three different cases:

$$N_F^1 = 200,\ R_G^1 = 1,\ N_F^2 = 40,\ R_G^2 = 5, \tag{59}$$

[3] Recall that by "topology" we mean a specific distribution of fundamental degrees of freedom among the geometry and the fields.

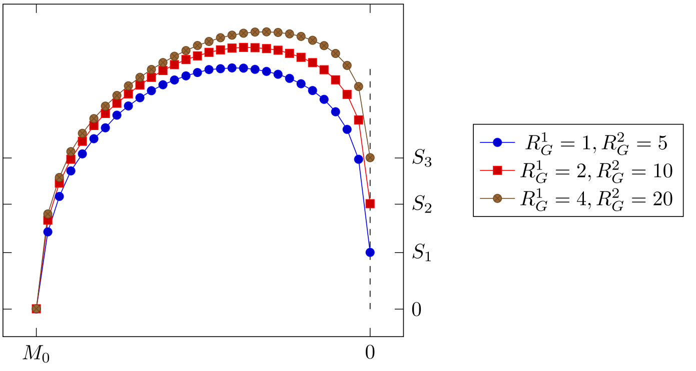

Fig. 3. Field entanglement entropy for two topologies and three cases. The figure shows that the more realizations of classical geometries are allowed, the higher is the resulting entanglement entropy. The residual entropies here are $S_1 = 0.77$, $S_2 = 1.43$, $S_3 = 2.06$.

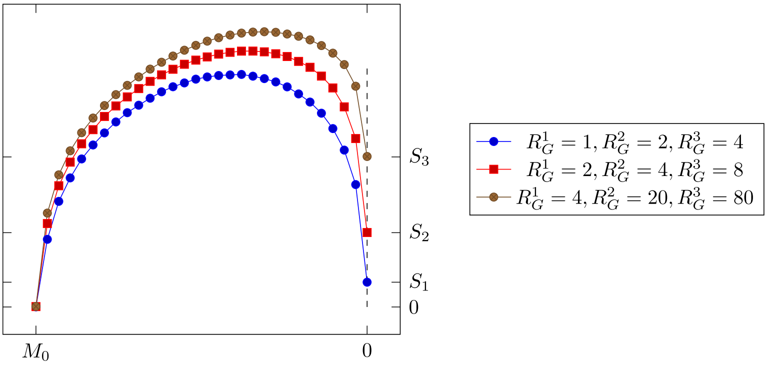

Fig. 4. Field entanglement entropy for three topologies and three cases. The figure shows that the more realizations of classical geometries are allowed, the higher is the resulting entanglement entropy. The residual entropies here are $S_1 = 0.34$, $S_2 = 1.02$, $S_3 = 2.06$.

$$N_F^1 = 200, R_G^1 = 2, N_F^2 = 40, R_G^2 = 10, \tag{60}$$

$$N_F^1 = 200, R_G^1 = 4, N_F^2 = 40, R_G^2 = 20. \tag{61}$$

The figure shows that the more realizations of classical geometries are allowed, the higher is the resulting entanglement entropy, that is the more the deviation from the Page curve (in the sense explained). The residual entropies are

$$S_1 = 0.77, \quad S_2 = 1.43, \quad S_3 = 2.06. \tag{62}$$

In Fig. 4 we choose three different topologies, $N_T = 3$, and three different cases with

$$N_F^1 = 120, N_F^2 = 60, N_F^3 = 30 \quad \text{and} \quad R_G^1 = 1, R_G^2 = 2, R_G^3 = 4, \tag{63}$$

$$N_F^1 = 120, N_F^2 = 60, N_F^3 = 30 \quad \text{and} \quad R_G^1 = 2, R_G^2 = 4, R_G^3 = 8, \tag{64}$$

$$N_F^1 = 200, N_F^2 = 40, N_F^3 = 10 \quad \text{and} \quad R_G^1 = 4, R_G^2 = 20, R_G^3 = 80, \tag{65}$$

In this case, the residual entropies are

$$S_1 = 0.34, \quad S_2 = 1.02, \quad S_3 = 2.06. \tag{66}$$

5. Conclusions

One possible interpretation of Bekenstein maximal bound on the number of degrees of freedom of any physical system, is that it is so because of the existence of entities more fundamental than the ones ordinarily deemed to be elementary [7,1–3]. We share this view here. Since at our energy scales such fundamental entities must organize themselves as quantum fields acting on classical spacetimes, we claim that such fundamental degrees of freedom should describe the state of both fields *and* geometry. In other words, fields and geometry are both emergent phenomena.

We investigate the kinematical and model independent effects on black hole evaporation of this "quasi-particle" view. We do so by constructing a simple model that allow us to produce a quantitative modifications of the Page curve of the entanglement entropy of Hawking radiation. We obtain that, at the end of the evaporation, the entanglement entropy stays finite, due to the unavoidable entanglement between geometry and fields. This can also be seen as the effect of more than one possible microscopic realization of the same emergent geometry. Indeed, even though we assume unitary evolution at the fundamental level, this inevitably leads to a reshuffling of the fundamental degrees of freedom, reflected at the emergent level as an entanglement between quantum fields and geometry. Therefore, part of the information associated to the quantum fields in the "phase" before the formation of the black hole is, in general, lost in the "phase" after the black hole has evaporated. Such features should be regarded as common to any theory of quantum gravity.

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